Perturbation Theory Of Dynamical Systems Arxiv

Reprint of classic reference work. Over 400 books have been published in the series Classics in Mathematics, many remain standard references for their subject. All books in this series are reissued in a new, inexpensive softcover edition to make them easily accessible to younger generations of students and researchers. "... The book has many good points: clear organization, historical notes and references at the end of every chapter, and an excellent bibliography. The text is well-written, at a level appropriate for the intended audience, and it represents a very good introduction to the basic theory of dynamical systems."

Stochastic Differential Equations have become increasingly important in modelling complex systems in physics, chemistry, biology, climatology and other fields. This book examines and provides systems for practitioners to use, and provides a number of case studies to show how they can work in practice.

This book deals with the theory of Poincaré--Birkhoff normal forms, studying symmetric systems in particular. Attention is focused on general Lie point symmetries, and not just on symmetries acting linearly. Some results on the simultaneous normalization of a vector field describing a dynamical system and vector fields describing its symmetry are presented and a perturbative approach is also used. Attention is given to the problem of convergence of the normalizing transformation in the presence of symmetry, with some other extensions of the theory. The results are discussed for the general case of dynamical systems and also for the specific Hamiltonian setting.

An introduction to certain aspects of developments in the modern theory of dynamics and simulation for a wide audience of scientifically literate readers. Unlike general texts on chaos theory and dynamical systems theory, this book follows the work on a specific problem at the very beginning of the modern era of dynamics, from its inception in 1954 through the early 1970s. It discusses such problems as the nonlinear oscillator simulation, the seminal discoveries at MIT in the early 1950s, the mathematical rediscovery of solitons in the late 1950s and the general problems of computability. In following these developments, the initial development of many of the now standard techniques of nonlinear modelling and numerical simulation are seen. No other text focuses so tightly and covers so completely one specific, pernicious problem at the heart of dynamics.

The second workshop on “Symmetry and Perturbation Theory” served as a forum for discussing the relations between symmetry and perturbation theory, and this put in contact rather different communities. The extension of the rigorous results of perturbation theory established for ODE's to the case of nonlinear evolution PDE's was also discussed: here a number of results are known, particularly in connection with (perturbation of) integrable systems, but there is no general frame as solidly established as in the finite-dimensional case. In aiming at such an infinite-dimensional extension, for which standard analytical tools essential in the ODE case are not available, it is natural to look primarily at geometrical and topological methods, and first of all at those based on exploiting the symmetry properties of the systems under study (both the unperturbed and the perturbed ones); moreover, symmetry considerations are in several ways basic to our understanding of integrability, i.e. finally of the unperturbed systems on whose understanding the whole of perturbation theory has unavoidably to rely.This volume contains tutorial, regular and contributed papers. The tutorial papers give students and newcomers to the field a rapid introduction to some active themes of research and recent results in symmetry and perturbation theory.

Mathematics of Complexity and Dynamical Systems is an authoritative reference to the basic tools and concepts of complexity, systems theory, and dynamical systems from the perspective of pure and applied mathematics. Complex systems are systems that comprise many interacting parts with the ability to generate a new quality of collective behavior through self-organization, e.g. the spontaneous formation of temporal, spatial or functional structures. These systems are often characterized by extreme sensitivity to initial conditions as well as emergent behavior that are not readily predictable or even completely deterministic. The more than 100 entries in this wide-ranging, single source work provide a comprehensive explication of the theory and applications of mathematical complexity, covering ergodic theory, fractals and multifractals, dynamical systems, perturbation theory, solitons, systems and control theory, and related topics. Mathematics of Complexity and Dynamical Systems is an essential reference for all those interested in mathematical complexity, from undergraduate and graduate students up through professional researchers.

This volume contains selected contributions from a very successful meeting on Number Theory and Dynamical Systems held at the University of York in 1987. There are close and surprising connections between number theory and dynamical systems. One emerged last century from the study of the stability of the solar system where problems of small divisors associated with the near resonance of planetary frequencies arose. Previously the question of the stability of the solar system was answered in more general terms by the celebrated KAM theorem, in which the relationship between near resonance (and so Diophantine approximation) and stability is of central importance. Other examples of the connections involve the work of Szemeredi and Furstenberg, and Sprindzuk. As well as containing results on the relationship between number theory and dynamical systems, the book also includes some more speculative and exploratory work which should stimulate interest in different approaches to old problems.

Many notions and results presented in the previous editions of this volume have since become quite popular in applications, and many of them have been “rediscovered” in applied papers. In the present 3rd edition small changes were made to the chapters in which long-time behavior of the perturbed system is determined by large deviations. Most of these changes concern terminology. In particular, it is explained that the notion of sub-limiting distribution for a given initial point and a time scale is identical to the idea of metastability, that the stochastic resonance is a manifestation of metastability, and that the theory of this effect is a part of the large deviation theory. The reader will also find new comments on the notion of quasi-potential that the authors introduced more than forty years ago, and new references to recent papers in which the proofs of some conjectures included in previous editions have been obtained. Apart from the above mentioned changes the main innovations in the 3rd edition concern the averaging principle. A new Section on deterministic perturbations of one-degree-of-freedom systems was added in Chapter 8. It is shown there that pure deterministic perturbations of an oscillator may lead to a stochastic, in a certain sense, long-time behavior of the system, if the corresponding Hamiltonian has saddle points. The usefulness of a joint consideration of classical theory of deterministic perturbations together with stochastic perturbations is illustrated in this section. Also a new Chapter 9 has been inserted in which deterministic and stochastic perturbations of systems with many degrees of freedom are considered. Because of the resonances, stochastic regularization in this case is even more important.

Mathematical models are often used to describe complex phenomena such as climate change dynamics, stock market fluctuations, and the Internet. These models typically depend on estimated values of key parameters that determine system behavior. Hence it is important to know what happens when these values are changed. The study of single-parameter deviations provides a natural starting point for this analysis in many special settings in the sciences, engineering, and economics. The difference between the actual and nominal values of the perturbation parameter is small but unknown, and it is important to understand the asymptotic behavior of the system as the perturbation tends to zero. This is particularly true in applications with an apparent discontinuity in the limiting behavior?the so-called singularly perturbed problems. Analytic Perturbation Theory and Its Applications includes a comprehensive treatment of analytic perturbations of matrices, linear operators, and polynomial systems, particularly the singular perturbation of inverses and generalized inverses. It also offers original applications in Markov chains, Markov decision processes, optimization, and applications to Google PageRank? and the Hamiltonian cycle problem as well as input retrieval in linear control systems and a problem section in every chapter to aid in course preparation.

This book provides an introduction to the theory of dynamical systems with the aid of the Mathematica® computer algebra package. The book has a very hands-on approach and takes the reader from basic theory to recently published research material. Emphasized throughout are numerous applications to biology, chemical kinetics, economics, electronics, epidemiology, nonlinear optics, mechanics, population dynamics, and neural networks. Theorems and proofs are kept to a minimum. The first section deals with continuous systems using ordinary differential equations, while the second part is devoted to the study of discrete dynamical systems.

Mathematical Tools for Physicists is a unique collection of 18 carefully reviewed articles, each one written by a renowned expert working in the relevant field. The result is beneficial to both advanced students as well as scientists at work; the former will appreciate it as a comprehensive introduction, while the latter will use it as a ready reference. The contributions range from fundamental methods right up to the latest applications, including: - Algebraic/ analytic / geometric methods - Symmetries and conservation laws - Mathematical modeling - Quantum computation The emphasis throughout is ensuring quick access to the information sought, and each article features: - an abstract - a detailed table of contents - continuous cross-referencing - references to the most relevant publications in the field, and - suggestions for further reading, both introductory as well as highly specialized. In addition, a comprehensive index provides easy access to the vast number of key words extending beyond the range of the headlines.

This volume provides a comprehensive review of multiple-scale dynamical systems. Mathematical models of such multiple-scale systems are considered singular perturbation problems, and this volume focuses on the geometric approach known as Geometric Singular Perturbation Theory (GSPT). It is the first of its kind that introduces the GSPT in a coordinate-independent manner. This is motivated by specific examples of biochemical reaction networks, electronic circuit and mechanic oscillator models and advection-reaction-diffusion models, all with an inherent non-uniform scale splitting, which identifies these examples as singular perturbation problems beyond the standard form. The contents cover a general framework for this GSPT beyond the standard form including canard theory, concrete applications, and instructive qualitative models. It contains many illustrations and key pointers to the existing literature. The target audience are senior undergraduates, graduate students and researchers interested in using the GSPT toolbox in nonlinear science, either from a theoretical or an application point of view. Martin Wechselberger is Professor at the School of Mathematics & Statistics, University of Sydney, Australia. He received the J.D. Crawford Prize in 2017 by the Society for Industrial and Applied Mathematics (SIAM) for achievements in the field of dynamical systems with multiple time-scales.

This book provides the mathematical foundations of the theory of hyperhamiltonian dynamics, together with a discussion of physical applications. In addition, some open problems are discussed. Hyperhamiltonian mechanics represents a generalization of Hamiltonian mechanics, in which the role of the symplectic structure is taken by a hyperkähler one (thus there are three Kähler/symplectic forms satisfying quaternionic relations). This has proved to be of use in the description of physical systems with spin, including those which do not admit a Hamiltonian formulation. The book is the first monograph on the subject, which has previously been treated only in research papers.

has been in the of a Symmetry major ingredient development quantum perturba tion and it is a basic of the of theory, ingredient theory integrable (Hamiltonian and of the the use in context of non Hamiltonian) systems; yet, symmetry gen eral is rather recent. From the of view of nonlinear perturbation theory point the use of has become dynamics, widespread only through equivariant symmetry bifurcation in this attention has been confined to linear even theory; case, mostly symmetries. in recent the and of methods for dif Also, theory practice symmetry years ferential has become and has been to a equations increasingly popular applied of the of the book Olver This by variety problems (following appearance [2621). with is and deals of nature theory deeply geometrical symmetries general (pro vided that described i.e. in this context there is are vector no they by fields), to limit attention to linear reason symmetries. In this look the basic tools of i.e. normal book we at perturbation theory, introduced Poincar6 about and their inter a forms (first by century ago) study action with with no limitation to linear ones. We focus on the most symmetries, basic fixed the and i.e. a setting, systems having point (at origin) perturbative around thus is local.

In this book we have developed the asymptotic analysis of nonlinear dynamical systems. We have collected a large number of results, scattered throughout the literature and presented them in a way to illustrate both the underlying common theme, as well as the diversity of problems and solutions. While most of the results are known in the literature, we added new material which we hope will also be of interest to the specialists in this field. The basic theory is discussed in chapters two and three. Improved results are obtained in chapter four in the case of stable limit sets. In chapter five we treat averaging over several angles; here the theory is less standardized, and even in our simplified approach we encounter many open problems. Chapter six deals with the definition of normal form. After making the somewhat philosophical point as to what the right definition should look like, we derive the second order normal form in the Hamiltonian case, using the classical method of generating functions. In chapter seven we treat Hamiltonian systems. The resonances in two degrees of freedom are almost completely analyzed, while we give a survey of results obtained for three degrees of freedom systems. The appendices contain a mix of elementary results, expansions on the theory and research problems.

Describes a approach to critical point theory and presents a whole new array of duality and perturbation methods.

A self-contained comprehensive introduction to the mathematical theory of dynamical systems for students and researchers in mathematics, science and engineering.

Creating some links between control feedback and biology modeling communities based on similarities in modeling, observing and perceiving alive structures, and analyzing interconnections between biological structures and subsystems was the main objective of this volume. In this context, biology systems need appropriate analysis tools due to their structure and hierarchy, complexity and environment interference, and we believe that these aspects may generate interesting research topics in control area. Indeed, several works, raising the potential impact of control developments to bring some beginning of answers in the context of biological systems, have been published in the recent years. The idea of this book was conceived in the context mentioned above with the objective to help in claiming many of the problems for control researchers, starting discussions and opening interactive debates between the control and biology communities, and, finally, to alert graduate students to the many interesting ideas at the frontier between control feedback theory and biology.

Symmetry and Perturbation Theory in Dynamical SystemsPerturbation theory for strongly perturbed dynamical systemsPerturbation Theory for Strongly Perturbed Dynamical SystemsIGeometric Singular Perturbation Theory Beyond the Standard FormSpringer Nature

This proceedings volume is devoted to the interplay of symmetry and perturbation theory, as well as to cognate fields such as integrable systems, normal forms, n-body dynamics and choreographies, geometry and symmetry of differential equations, and finite and infinite dimensional dynamical systems. The papers collected here provide an up-to-date overview of the research in the field, and have many leading scientists in the field among their authors, including: D Alekseevsky, S Benenti, H Broer, A Degasperis, M E Fels, T Gramchev, H Hanssmann, J Krashil"shchik, B Kruglikov, D Krupka, O Krupkova, S Lombardo, P Morando, O Morozov, N N Nekhoroshev, F Oliveri, P J Olver, J A Sanders, M A Teixeira, S Terracini, F Verhulst, P Winternitz, B Zhilinskii. Sample Chapter(s). Foreword (101 KB). Chapter 1: Homogeneous Bi-Lagrangian Manifolds and Invariant Monge-Ampere Equations (415 KB). Contents: On Darboux Integrability (I M Anderson et al.); Computing Curvature without Christoffel Symbols (S Benenti); Natural Variational Principles (D Krupka); Fuzzy Fractional Monodromy (N N Nekhoroshev); Emergence of Slow Manifolds in Nonlinear Wave Equations (F Verhulst); Complete Symmetry Groups and Lie Remarkability (K Andriopoulos); Geodesically Equivalent Flat Bi-Cofactor Systems (K Marciniak); On the Dihedral N-Body Problem (A Portaluri); Towards Global Classifications: A Diophantine Approach (P van der Kamp); and other papers. Readership: Researchers and students (graduate/advanced undergraduates) in mathematics, applied mathematics, physics and nonlinear science.

This book aims to present a new approach called Flow Curvature Method that applies Differential Geometry to Dynamical Systems. Hence, for a trajectory curve, an integral of any n-dimensional dynamical system as a curve in Euclidean n-space, the curvature of the trajectory — or the flow — may be analytically computed. Then, the location of the points where the curvature of the flow vanishes defines a manifold called flow curvature manifold. Such a manifold being defined from the time derivatives of the velocity vector field, contains information about the dynamics of the system, hence identifying the main features of the system such as fixed points and their stability, local bifurcations of codimension one, center manifold equation, normal forms, linear invariant manifolds (straight lines, planes, hyperplanes).In the case of singularly perturbed systems or slow-fast dynamical systems, the flow curvature manifold directly provides the slow invariant manifold analytical equation associated with such systems. Also, starting from the flow curvature manifold, it will be demonstrated how to find again the corresponding dynamical system, thus solving the inverse problem.

To understand multiscale phenomena, it is essential to employ asymptotic methods to construct approximate solutions and to design effective computational algorithms. This volume consists of articles based on the AMS Short Course in Singular Perturbations held at the annual Joint Mathematics Meetings in Baltimore (MD). Leading experts discussed the following topics which they expand upon in the book: boundary layer theory, matched expansions, multiple scales, geometric theory, computational techniques, and applications in physiology and dynamic metastability. Readers will find that this text offers an up-to-date survey of this important field with numerous references to the current literature, both pure and applied.

Mathematicians often face the question to which extent mathematical models describe processes of the real world. These models are derived from experimental data, hence they describe real phenomena only approximately. Thus a mathematical approach must begin with choosing properties which are not very sensitive to small changes in the model, and so may be viewed as properties of the real process. In particular, this concerns real processes which can be described by means of ordinary differential equations. By this reason different notions of stability played an important role in the qualitative theory of ordinary differential equations commonly known nowdays as the theory of dynamical systems. Since physical processes are usually affected by an enormous number of small external fluctuations whose resulting action would be natural to consider as random, the stability of dynamical systems with respect to random perturbations comes into the picture. There are differences between the study of stability properties of single trajectories, i. e. , the Lyapunov stability, and the global stability of dynamical systems. The stochastic Lyapunov stability was dealt with in Hasminskii [Has]. In this book we are concerned mainly with questions of global stability in the presence of noise which can be described as recovering parameters of dynamical systems from the study of their random perturbations. The parameters which is possible to obtain in this way can be considered as stable under random perturbations, and so having physical sense. -1- Our set up is the following.

This book which focusses on mechanics, waves and statistics, describes recent developments in the application of differential geometry, particularly symplectic geometry, to the foundations of broad areas of physics. Throughout the book, intuitive descriptions and diagrams are used to elucidate the mathematical theory. It develops a coordinate-free framework for perturbation theory and uses this to show how underlying symplectic structures arise from physical asymptotes. It describes a remarkable parity between classical mechanics which arises asymptotically from quantum mechanics and classical thermodynamics which arises asymptotically from statistical mechanics. Included here is a section with one hundred unanswered questions for further research.

This book is an extended version of lectures given by the ?rst author in 1995–1996 at the Department of Mechanics and Mathematics of Moscow State University. We believe that a major part of the book can be regarded as an additional material to the standard course of Hamiltonian mechanics. In comparison with the original Russian 1 version we have included new material, simpli?ed some proofs and corrected m- prints. Hamiltonian equations ?rst appeared in connection with problems of geometric optics and celestial mechanics. Later it became clear that these equations describe a large classof systemsin classical mechanics,physics,chemistry,and otherdomains. Hamiltonian systems and their discrete analogs play a basic role in such problems as rigid body dynamics, geodesics on Riemann surfaces, quasi-classic approximation in quantum mechanics, cosmological models, dynamics of particles in an accel- ator, billiards and other systems with elastic re?ections, many in?nite-dimensional models in mathematical physics, etc. In this book we study Hamiltonian systems assuming that they depend on some parameter (usually?), where for?= 0 the dynamics is in a sense simple (as a rule, integrable). Frequently such a parameter appears naturally. For example, in celestial mechanics it is accepted to take? equal to the ratio: the mass of Jupiter over the mass of the Sun. In other cases it is possible to introduce the small parameter ar- ?cially.

Perturbations: Theory and Methods gives a thorough introduction to both regular and singular perturbation methods for algebraic and differential equations. Unlike most introductory books on the subject, this one distinguishes between formal and rigorous asymptotic validity, which are commonly confused in books that treat perturbation theory as a bag of heuristic tricks with no foundation. The meaning of "uniformity" is carefully explained in a variety of contexts. All standard methods, such as rescaling, multiple scales, averaging, matching, and the WKB method are covered, and the asymptotic validity (in the rigorous sense) of each method is carefully proved. First published in 1991, this book is still useful today because it is an introduction. It combines perturbation results with those known through other methods. Sometimes a geometrical result (such as the existence of a periodic solution) is rigorously deduced from a perturbation result, and at other times a knowledge of the geometry of the solutions is used to aid in the selection of an effective perturbation method. Dr. Murdock's approach differs from other introductory texts because he attempts to present perturbation theory as a natural part of a larger whole, the mathematical theory of differential equations. He explores the meaning of the results and their connections to other ways of studying the same problems.

Asymptotical problems have always played an important role in probability theory. In classical probability theory dealing mainly with sequences of independent variables, theorems of the type of laws of large numbers, theorems of the type of the central limit theorem, and theorems on large deviations constitute a major part of all investigations. In recent years, when random processes have become the main subject of study, asymptotic investigations have continued to playa major role. We can say that in the theory of random processes such investigations play an even greater role than in classical probability theory, because it is apparently impossible to obtain simple exact formulas in problems connected with large classes of random processes. Asymptotical investigations in the theory of random processes include results of the types of both the laws of large numbers and the central limit theorem and, in the past decade, theorems on large deviations. Of course, all these problems have acquired new aspects and new interpretations in the theory of random processes.

Based on the recent NATO Advanced Study Institute "Chaotic Worlds: From Order to Disorder in Gravitational N-Body Dynamical Systems", this state of the art textbook, written by internationally renowned experts, provides an invaluable reference volume for all students and researchers in gravitational n-body systems. The contributions are especially designed to give a systematic development from the fundamental mathematics which underpin modern studies of ordered and chaotic behaviour in n-body dynamics to their application to real motion in planetary systems. This volume presents an up-to-date synoptic view of the subject.

This new text/reference is an excellent resource for the foundations and applications of control theory and nonlinear dynamics. All graduates, practitioners, and professionals in control theory, dynamical systems, perturbation theory, engineering, physics and nonlinear dynamics will find the book a rich source of ideas, methods and applications. With its careful use of examples and detailed development, it is suitable for use as a self-study/reference guide for all scientists and engineers.